

Numerical Analysis and Simulation of Solute Transport in Heterogeneous Porous Medium via Crank-Nicolson Finite Difference Scheme

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Abstract

Keywords:

Solute Transport, Fluctuating boundary flux, First-order attenuation, Zero-order generation, Porous Medium, Velocity, Finite difference strategy, Crank-Nicolson Method.

A Numerical framework is developed to investigate one-dimensional solute migration in heterogeneous porous media under variable transport conditions. By employing the Crank-Nicolson finite difference strategy - noted for its second order accuracy and inherent stability - that quantifies concentration behavior influenced by fluctuating groundwater velocities and non-uniform dispersion coefficients. By utilizing this numerical strategy, the framework effectively characterizes solute dynamics for space-time dependent decay and production parameter that vary sinusoidally $|1 + \sin(\lambda x t)|$ and exponentially $\exp(-\lambda x t)$ with position and time. The results of this work reveal the spatial-temporal evolution of the contaminant plume and demonstrate the profound effects of variable hydrodynamic dispersions on solute migration within the given heterogeneous porous domain.

Introduction

Increasing industrial discharge, agricultural intensification and improper waste management along with anthropogenic activities have raised critical environmental concerns for groundwater contamination. With the rise in human-induced environmental activities it has become difficult to predict and understand solute migration in waterbodies. Mathematical modelling and concepts of physics emerge out as one of the most powerful tools that can act as framework for analyzing contaminant transport processes with respect to different physical parameters. These models serve as a predictive apparatus for examining the variation of contaminant migration patterns across both spatial temporal dimensions. The transport of solutes and contaminant analysis in groundwater systems is fundamentally controlled by coupled advection, dispersion, and diffusion processes occurring within heterogeneous porous media. Over the past several decades, a broad spectrum of analytical and numerical frameworks has been established to characterize the migration of conservative solute across both bounded and unbounded porous formations.

Many researchers have proposed analytical solutions of the advection dispersion equation (ADE) under different physical conditions. In 1995, J. Logan and V. Zlotnic derived an analytical solution decaying in a particular system for groundwater table fluctuations over time [1]. D. Barry and G. Sposito have given solution for predicting to find transport equation where velocity and dispersion are based on time [2]. H. Kangle, Genuchten and Renduo provided an exact solution of contaminant movement by dispersion with scale dependent [3]. R. Yadav, R. Vinda and N. Kumar formulated solution of one-dimensional ADE with fluctuate flow [4]. Similar study was carried out for the one-dimensional transport problem were given by V. Srinivasan and P. Clement. They used varying initial condition and exponential decays [5]. Various other analytical studies addressing one-dimensional solute transport problems in finite and semi-infinite porous media have also been reported in the literature [6]. However most analytical solutions are obtained under idealized assumptions such as homogeneous porous media, simplified flow conditions, and constant or uniform dispersion coefficients, which limit their applicability to complex real-world transport problems [7]. In practical scenarios, the controlling advection diffusion equation is generally nonlinear, which makes it difficult to obtain analytical solutions for the problem; therefore numerical solution of advection dispersion equation (ADE) plays a crucial role in finding approximate solutions of the transport problems in aquifer systems. To overcome the limitations of analytical approaches, several numerical methods have been developed to solve the ADE under more realistic conditions. J. Tamora and C. Wadham proposed a theoretical solution of transport equation in which pollutant disperse in circular pattern [8]. A. Elfeki expanded a numerical model based on finite difference schemes to simulate transient groundwater flow within geological structures [9]. Algorithm for aquatic environment contaminant was solve by B. Agosto and M. Bamingbola [10]. A hydrodynamic model was developed by Pochai for solvent diffusion in a uniform reservoir [11]. G. Gurarslan and H. Karahan derived solutions of the ADE using finite difference schemes to estimate pollutant transport in river systems [12].

Among the available numerical methods, Crank-Nicolson method (CN method) is widely recognized for its inherent stability and second order precision when computing solutions of transient partial differential equations. The proposed method provides concentration at different positions which spreads exponentially at each time step [16]. The properties of CN make it particularly suitable for solving transport equations such as ADE where an accurate prediction of concentration distribution is required over extended spatial and temporal domains. The inherent physical diversity of porous systems significantly affects solute transport leading to variations in groundwater flow dynamics [13]. Assumption has been taken that the seepage velocity varied sinusoidally and exponentially with time and space. First-order attenuation and zero-order generation parameters are included in the model while assuming a linear dependency where the dispersion coefficient varies proportionally to groundwater velocity. The porous formation is treated as heterogeneous where intrinsic variations in pore distribution and size distribution dictate the spatial dependency of transport coefficients such fluctuations in velocity and dispersion are considered as fundamental characteristics of natural aquifer systems. Within the longitudinal coordinate system along positive x-axis the domain away from the origin and another condition is specified at the domain boundary. Numerical results are obtained under conditions of both constant and time-dependent point source configuration. Solute transport behavior under variable dispersion conditions are illustrated using realistic data from published literature. The transport dynamics in this work are defined by the one-dimensional advection dispersion framework and computed using specialized numerical methods. The resulting data are synthesized in forms of table and visually demonstrated through graphs developed within the MATLAB environment.

Theoretical Framework and Governing Transport Equation

The present formulation examines one-dimensional migration of a conservative contaminant finite heterogeneous porous medium, where the solute source is introduced at $x = 0$ within the longitudinal domain as illustrated in the schematic representation in Figure 1.

The migration of solute within a porous matrix governed by two simultaneous phenomena: Advection and hydrodynamic dispersion. Advection refers to the transport of solutes resulting from the bulk movements of groundwater, whereas dispersion emerges from a combination of mechanical mixing and molecular diffusion within the complex pore architecture. This processes are mathematically integrated into the advection-dispersion equation (ADE) second-order parabolic partial differential equation that also accounts for reaction and source terms. The governing equation derived with the help of Darcy's law, the principle of mass conservation and Fick's law of diffusion in this work is expressed as follows [14-15]:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left(D(x,t) \frac{\partial C}{\partial x} - u(x,t) C \right) - \gamma C + \mu \quad (1)$$

The formulation parameters involved in the governing equation with their corresponding dimensions are as follows: C represents solute concentration [ML^{-3}] in the liquid phase. x and t denote spatial coordinal [L] and time [T] variable, D is the hydrodynamic dispersion coefficient [L^2T^{-1}], u is the groundwater velocity [LT^{-1}], γ and μ are the first-order decay [T^{-1}] and zero-order production [$ML^{-3}T^{-1}$], λ parameter [$L^{-1}T^{-1}$] respectively.

For numerical implementation, the present study adopts a variable co-efficient advection-dispersion formulation in which the effects of medium heterogeneity are incorporated through the spatial-temporal dependence of the transport coefficients $D(x,t)$ and $u(x,t)$. Accordingly the following variable co-efficient transport equation is employed for the CN discretization.

$$\frac{\partial C}{\partial t} = D(x,t) \frac{\partial^2 C}{\partial x^2} - u(x,t) \frac{\partial C}{\partial x} - \gamma C + \mu \quad (2)$$

The co-efficient functions are evaluated locally at each computational node thereby preserving the influence of heterogeneity on solute migration while maintaining computationally traceable numerical framework.

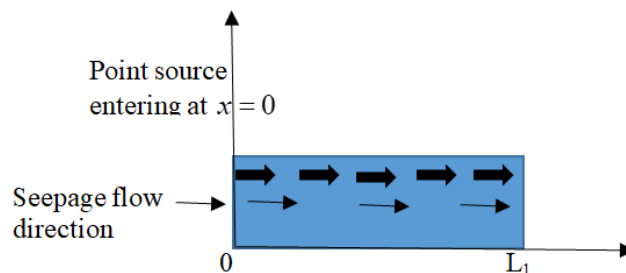


Figure 1: Conceptual layout of proposed model

Uniform Source Boundary Condition

A contaminant is introduced at a constant rate into finite ($0 \leq x(m) \leq 5$), non-homogeneous porous system at a position $x = 0$. The transport is analysed along the direction of flow from the injection site toward the boundary $x = L_1$. The initial state of the system is characterised by the background concentration with the highest pollutant levels observed at a source, which is further attenuated by distance and time throughout the transport process.

The mathematical framework for this problem governed by the ADE is defined by the following set of initial and boundary conditions:

$$\text{Initial state } (t = 0): C(x, t) = C_i e^{-ax}; 0 < x < L_1, \quad (3)$$

$$\text{Inlet boundary } (x = 0): C(x, t) = C_0; t > 0, \quad (4)$$

$$\text{Outlet boundary } (x = L_1): \frac{\partial C(x, t)}{\partial x} = \frac{u}{D} C; t \geq 0, \quad (5)$$

To account for the medium's heterogeneity, the transport parameters including seepage velocity (u), hydrodynamic dispersion (D), Decay coefficient (γ) and the production rate (μ) are modelled to be function of space and time. Which are assumed to follow unified functional dependence $g(x, t)$ and are represented as

$$\left. \begin{aligned} u(x, t) &= u_0 g(x, t) \\ D(x, t) &\propto u(x, t) \Rightarrow D(x, t) = D_0 g(x, t) \\ \gamma(x, t) &= \gamma_0 g(x, t) \\ \mu(x, t) &= \mu_0 g(x, t) \end{aligned} \right\} \quad (6)$$

where u_0 , D_0 , γ_0 and μ_0 signify the reference values for velocity, dispersion first-order decay, and zero-order production, respectively. The function $g(x, t)$ serves as a dimensionless scaling that satisfies constrain $g(x, t) = 1$ at $x = 0$ or $t = 0$. Two distinct mathematical forms for this scaling factor is taken into account which are:

$$\left. \begin{aligned} i) g(x, t) &= e^{-\lambda x t} \\ ii) g(x, t) &= |1 + \sin(\lambda x t)| \end{aligned} \right\} \quad (7)$$

Utilising a unified scaling function $g(x, t)$ serves as strategic modelling simplification to analyse spatial-temporal heterogeneity within a consolidated mathematical framework. While groundwater velocity u and dispersion D are physically coupled via pore-scale dynamics, reaction kinetics and production inputs (μ) typically follow independent pathways in natural systems. Consequently this framework is presented as an idealized benchmark model designed to explore synchronised multi-parameter variations rather than to replicate site-specific hydro-chemical profiles. The stylized coupling effectively approximates a specialized environment where reactive mineral distributions, localised microbial activity or solute generation rates scale directly with fluid flux and pore network geometry in highly correlated heterogeneous media thereby maintaining numerical tractability.

The reformulation of eq. (1) is done by substituting parameter relationships into the primary governing equation,

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left(D_0 g(x, t) \frac{\partial C}{\partial x} - u_0 g(x, t) C \right) - (\gamma_0 C - \mu_0) g(x, t) \quad (8)$$

The simulation of the conceptual layout follows a specific set of Initial and boundary constrains are as:

$$C(x, t) = C_i e^{-ax}; t = 0, 0 < x < L_1, \quad (9)$$

$$C(x, t) = C_0; t > 0, x = 0, \quad (10)$$

$$\frac{\partial C(x, t)}{\partial x} = \frac{u}{D} C; t \geq 0, x = L_1 \quad (11)$$

Time Dependent Source Condition

The source boundary flux at $x = 0m$ may vary temporally and is modelled using a mixed (third-type) boundary condition. A zero-flux condition was applied along the longitudinal flow for all simulations.

$$-D \frac{\partial C(x,t)}{\partial x} + uC(x,t) = u_0 C_0 \{1 + \sin(mt)\}; t > 0, x = 0 \quad (12)$$

$$\frac{\partial C(x,t)}{\partial x} = 0; t \geq 0, x = L_1 \quad (13)$$

Using eq. (5) in eq. (8) we get,

$$-D_0 g(x,t) \frac{\partial C}{\partial x} + u_0 g(x,t) C(x,t) = u_0 C_0 \{1 + \sin(mt)\}; t > 0, x = 0 \quad (14)$$

$$\frac{\partial C(x,t)}{\partial x} = 0; t \geq 0, x = L_1 \quad (15)$$

The Crank Nicolson Method

In the present formulation, temporal discretization is performed with respect to time t with the help of forward finite difference approximation, while the spatial derivatives are evaluated along the longitudinal x -direction using second order central difference approximation in eq. (2). Let C_i^j denote the numerical concentration value at spatial node x_i and time level t_j , then define the approximate concentration as $C(x_i, t_j)$.

$$\frac{\partial C}{\partial t} = \frac{C_i^{j+1} - C_i^j}{\Delta t} \quad (16)$$

$$\frac{\partial C}{\partial x} = \frac{C_{i+1}^j - C_{i-1}^j}{2\Delta x} \quad (17)$$

$$\frac{\partial^2 C}{\partial x^2} = \frac{C_{i-1}^j - 2C_i^j + C_{i+1}^j}{(\Delta x)^2} \quad (18)$$

The implicit Crank-Nicolson numerical framework approximations of the governing equation by considering eqns. (17) and (18) are estimated by taking mean the discretized spatial terms between successive time levels j and $(j+1)$. The numerical scheme for Crank-Nicolson is mentioned here.

$$\begin{aligned} \frac{C_i^{j+1} - C_i^j}{\Delta t} = & \frac{D_i}{2} \left\{ \left[\frac{C_{i-1}^{j+1} - 2C_i^{j+1} + C_{i+1}^{j+1}}{(\Delta x)^2} + \frac{C_{i-1}^j - 2C_i^j + C_{i+1}^j}{(\Delta x)^2} \right] \right\} \\ & - \frac{u_i}{2} \left\{ \left[\frac{C_{i+1}^{j+1} - C_{i-1}^{j+1}}{2\Delta x} + \frac{C_{i+1}^j - C_{i-1}^j}{2\Delta x} \right] \right\} - \gamma_i \{C_i^{j+1} + C_i^j\} + \mu_i \end{aligned} \quad (19)$$

After arranging the unknown concentration terms at the $(j+1)$ -th time level L.H.S and j -th time level on R.H.S eq. (20) is obtained.

$$\begin{aligned} & -(rD_i + u_i s)C_{i-1}^{j+1} + \left(1 + 2rD_i + \frac{\Delta t}{2}\gamma_i\right)C_i^{j+1} + (-rD_i + u_i s)C_{i+1}^{j+1} = \\ & (rD_i + u_i s)C_{i-1}^j + \left(1 - 2rD_i - \frac{\Delta t}{2}\gamma_i\right)C_i^j + (rD_i - u_i s)C_{i+1}^j + \mu_i \Delta t \end{aligned} \quad (20)$$

where, $r = \frac{\Delta t}{2(\Delta x)^2}$ and $s = \frac{\Delta t}{4\Delta x}$

On further simplification, the implicit Crank-Nicolson discretization of eq. (20) yields a system of linear equations that can be written in a matrix system as

$$AC^{j+1} = BC^j + S \quad (21)$$

Where C^{j+1} and C^j denote the concentration vectors at the $(j+1)$ -th and j -th time levels, respectively, and S represents the source term vector.

$$C^{j+1} = \begin{bmatrix} C_1^{j+1} \\ C_2^{j+1} \\ \vdots \\ C_M^{j+1} \end{bmatrix}, \quad C^j = \begin{bmatrix} C_1^j \\ C_2^j \\ \vdots \\ C_M^j \end{bmatrix},$$

where M is the number of interior spatial nodes.

The co-efficient matrices A and B are tridiagonal and are given by

$$A = \begin{bmatrix} b_1 & c_1 & 0 & \dots & 0 \\ a_2 & b_2 & c_2 & \dots & 0 \\ 0 & a_3 & b_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & c_{M-1} \\ 0 & 0 & 0 & a_M & b_M \end{bmatrix} \quad (22)$$

$$B = \begin{bmatrix} e_1 & f_1 & 0 & \dots & 0 \\ d_2 & e_2 & f_2 & \dots & 0 \\ 0 & d_3 & e_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & f_{M-1} \\ 0 & 0 & 0 & d_M & e_M \end{bmatrix} \quad (23)$$

where

$$\begin{aligned} a_i &= -(rD_i + u_i s) \\ b_i &= 1 + 2rD_i + \frac{\Delta t}{2} \gamma_i \\ c_i &= -rD_i + u_i s \\ d_i &= rD_i + u_i s \\ e_i &= 1 - 2rD_i - \frac{\Delta t}{2} \gamma_i \\ f_i &= rD_i - u_i s \end{aligned}$$

The source term vector is $S = \begin{bmatrix} \mu_1 \Delta t \\ \mu_2 \Delta t \\ \vdots \\ \mu_M \Delta t \end{bmatrix}$ (24)

The resulting tridiagonal system is solved iteratively at each step to obtain the concentration distribution within the finite heterogeneous porous medium. The proposed problem is solved numerically for both cases using Crank-Nicolson method and the results are presented graphically as well as in tabular form generated by MATLAB.

Result And Discussion

The numerical behavior of the advection dispersion model was evaluated across finite domain *i.e.* from $x=0$ to $L_1=5$ using the Crank-Nicolson finite difference formulation. The transport behavior was analyzed at different spatial locations and time instances under both uniform and time dependent source conditions.

The spatial and temporal units are define in meters and days respectively. For all simulations, A baseline configuration is established using an initial seepage velocity ($u_0 = 0.12 \text{ m/day}$), initial dispersion coefficient ($D_0 = 0.20 \text{ m}^2/\text{day}$) and heterogeneity factor ($m = 0.01 \text{ day}^{-1}$). Additional reactions constants included a first-order decay rate ($\gamma_0 = 0.0020 \text{ day}^{-1}$), zero-order production term ($\mu_0 = 0.0025 \text{ kg/m}^3 \text{ day}$), time $t = 4 \text{ day}$, $\alpha = 0.015 \text{ meter}^{-1}$ and $\lambda = 0.1 \text{ meter}^{-1} \text{ day}^{-1}$.

The numerical trends for both steady and fluctuating point sources are illustrated through the series of line points in figure (a) through figure (d). Figure (a) and Figure (b) depict the spatial distribution of contaminant concentration corresponding to variable

boundary flux at two different time levels ($t = 2$ days and $t = 4$ days) obtained through the numerical solution of the governing ADE respectively. The computational results demonstrate a positive correlation between time and solute penetration where concentration levels increase as the contaminant plume propagates further within the formation over time for both the velocity forms. The sensitivity of the system to hydrodynamic dispersion is further explored in figure (c) and figure (d) comparing results for $D_0 = 0.20 \text{ m}^2 / \text{day}$ and $D_0 = 0.40 \text{ m}^2 / \text{day}$. The contaminant concentration is slightly lower near the source boundary for at $x = 0 \text{ m}$ where dispersion coefficient is generally high and vice versa. However, as the distance from source increases this behavior generally reverses due to enhanced spreading of contaminant plume. The parameters influencing solute migration within the porous aquifer domains are constant for all cases. The concentration profiles show a characteristic decrease as the distance from boundary ($x = 0 \text{ m}$) increase, driven by the synergistic effects of advection, dispersion and reactive decay. Structural similarity in overall transport patterns for both exponential and sinusoidal velocity forms may appear but the exponential case displays a marginally more attenuation of the solute. The concentration profiles tends towards steady state condition and approach very small values after a certain distance.

Table 1 summarizes the concentration values for different spatial domains at different time instances with $D_0 = 0.20 \text{ m}^2 / \text{day}$. The results indicate that concentration decreases rapidly as distance increases due to dispersion and decay effects. Furthermore the concentration values at specific spatial locations are significantly higher at $t = 4$ days when compared to $t = 2$ days within the porous media as visually illustrated in figure (a) and (b).

The findings illustrated in figures (c) and (d) and summarized in table 2, demonstrate that while solute concentration generally decrease with distance. The influence of hydrodynamic dispersion exhibits a distinct spatially dependent behavior. A higher reference dispersion coefficient ($D_0 = 0.40 \text{ m}^2 / \text{day}$) yields comparatively higher concentration values in the mid to far spatial domain ($x \geq 2.00 \text{ m}$) because the contaminant plume spreads more widely along flow direction. However this phenomena introduces a critical physical trade-off involving competing effects of local transport and downstream transport. At $x = 0 \text{ m}$ near the injection zone accelerated mechanical mixing trends to reduce localized concentration levels by distributing mass over larger spatial extent. Because a dirichlet (first-type) boundary condition rigidly constrains the inlet boundary to a constant value ($C = C_0 = 1.00$ at $x = 0$), this local dilution effect does not alter the absolute origin value itself. Instead the trade-off manifests immediately downstream from the inlet node, where enhanced longitudinal mixing rapidly drives mass forward generating the distinct breakthrough gradients observed across the domain interior.

Table1: Temporal evolution of solute levels at fixed dispersion coefficient $D_0 = 0.20 \text{ m}^2 / \text{day}$

Position ($\mathbf{x}$)	Uniform input				Varying input			
	Exponential		Sinusoidal		Exponential		Sinusoidal	
	$t = 2$	$t = 4$	$t = 2$	$t = 4$	$t = 2$	$t = 4$	$t = 2$	$t = 4$
1.00	0.331249	0.511478	0.384704	0.624478	0.091555	0.192125	0.110394	0.258338
2.00	0.042434	0.131742	0.078803	0.299628	0.018739	0.046423	0.027854	0.111617
3.00	0.013821	0.023901	0.020988	0.119797	0.013395	0.017088	0.016730	0.050185
4.00	0.012875	0.014594	0.016434	0.050020	0.012874	0.014486	0.016263	0.030994
5.00	0.012594	0.013907	0.016419	0.033912	0.012594	0.013907	0.016411	0.027230

Table 2: Concentration Distribution under dispersion variation at time $t = 4 \text{ day}$

Position ($\mathbf{x}$)	Uniform input				Varying input			
	Exponential		Sinusoidal		Exponential		Sinusoidal	
	$D_0 = 0.20$	$D_0 = 0.40$	$D_0 = 0.20$	$D_0 = 0.40$	$D_0 = 0.20$	$D_0 = 0.40$	$D_0 = 0.20$	$D_0 = 0.40$
1.00	0.511478	0.614671	0.624478	0.717109	0.192125	0.186708	0.258338	0.244584
2.00	0.131742	0.263235	0.299628	0.453548	0.046423	0.070874	0.111617	0.140074
3.00	0.023901	0.073035	0.119797	0.259157	0.017088	0.025285	0.050185	0.079636
4.00	0.014594	0.020650	0.050020	0.144078	0.014486	0.015341	0.030994	0.049542
5.00	0.013907	0.014548	0.033912	0.105758	0.013907	0.014095	0.027230	0.040444

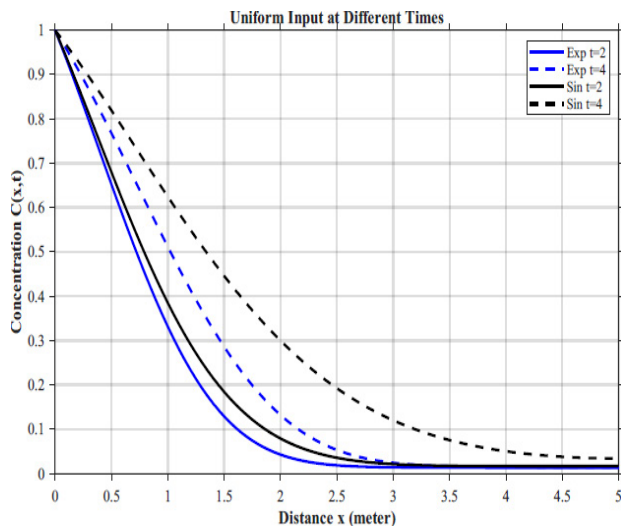


Figure (a): Concentration distribution for varying times at dispersion coefficient $D_0 = 0.20 \text{ m}^2 / \text{day}$ with uniform input point source. (Table 1)

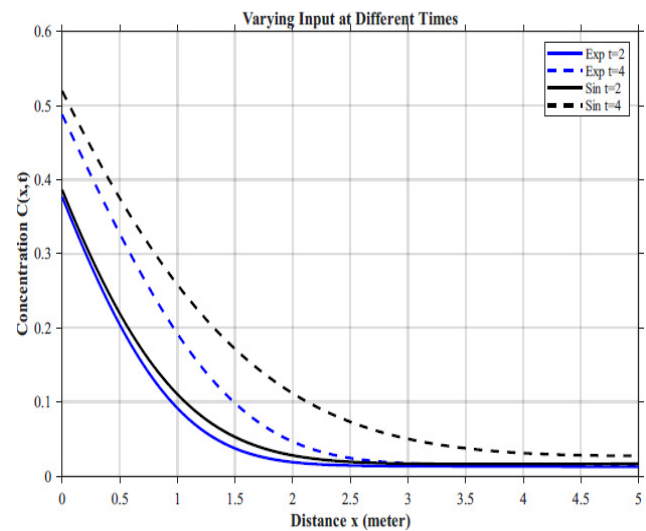


Figure (b): Concentration distribution for varying times at dispersion coefficient $D_0 = 0.20 \text{ m}^2 / \text{day}$ with varying input point source. (Table 1)

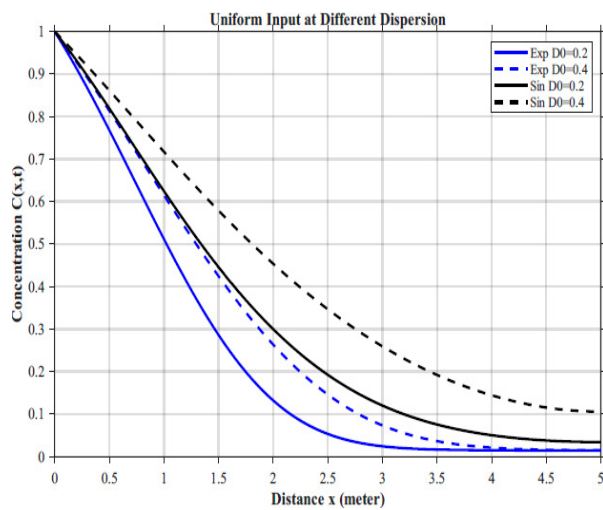


Figure (c): Concentration distribution for varying dispersion coefficients at time $t = 4 \text{ day}$ with uniform input point source. (Table 2)

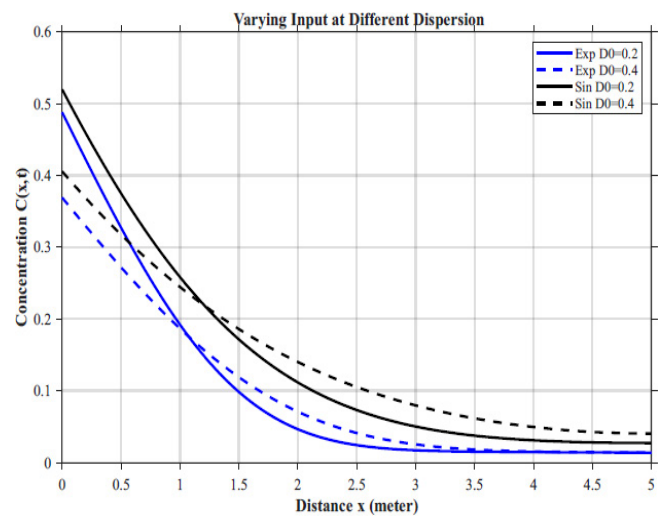


Figure (d): Concentration distribution for varying dispersion coefficients at time $t = 4 \text{ day}$ with varying input point source. (Table 2)

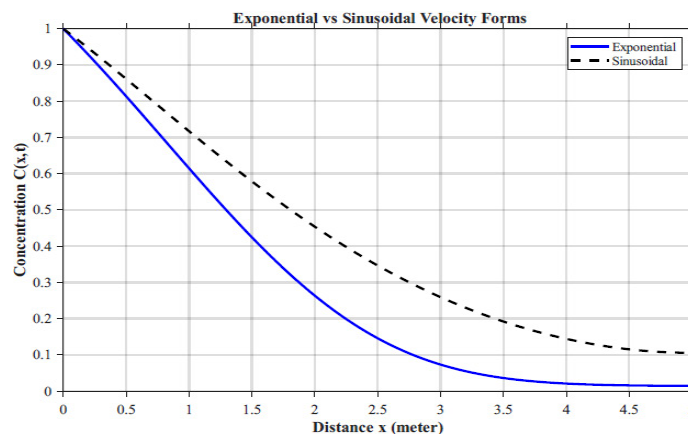


Figure (e): Concentration distribution for exponential and sinusoidal velocity at time and $D_0 = 0.20 \text{ m}^2 / \text{day}$.

Table 3: Outlet concentration comparison at $x = 5m$ and $t = 4$ days

Velocity forms	Concentration
Exponential	0.014548
Sinusoidal	0.105758
Absolute Difference = 0.091210	
Ratio(sin/exp) = 7.269775	
Percentage difference = 626.98%	

The figure (e) presents the analysis of the exponential and sinusoidal velocity variation forms. Both velocity forms produced overall similar concentration patterns at different positions in the medium which reveals that concentration decreases gradually along the spatial domain. The higher concentration level is absorbed by sinusoidal velocity field at farther distances when compared to the exponential velocity field.

The tabular comparison (Table 3) shows that outlet concentration ($x=5$) is higher for sinusoidal velocity as compared to exponential velocity, indicating more efficient contaminant transport. The sinusoidal velocity field enhances transport because periodic velocity increases creating stronger advection allowing contaminant to propagate further than the compared to continuously decreasing exponential field.

Conclusion

A numerical investigations in this work provides the framework for analyzing one-dimensional solute migration within finite heterogeneous porous environments and qualitative results were obtained using MATLAB for the same. These numerical results are useful for mapping the distribution of solute levels, throughout the domain providing a continuous profile of plume's evolution across space and time. The obtained graphs show that solute intensity is greatly influenced by the combined effects of spatial and temporal transport variations. The applied model can be extended to real-field waterbodies data for predictive environmental monitoring.

References

- Logan, J.D., Zlotnik, V. (1995). The convection diffusion equation with periodic boundary conditions. *Applied Mathematics Letters*, 8(3): 55-61. [https://doi.org/10.1016/0893-9659\(95\)00030-t](https://doi.org/10.1016/0893-9659(95)00030-t)
- Barry, D.A., Sposito, G. (1989). Analytical solution of a convection-dispersion model with time-dependent transport coefficients. *Water Resources Research*, 25(12): 2407-2416. <https://doi.org/10.1029/wr025i012p02407>
- Kangle, H., van Genuchten, M.T., Renduo, Z. (1996). Exact solutions for one-dimensional transport with asymptotic scale-dependent dispersion. *Applied Mathematical Modelling*, 20(4): 298-308. [https://doi.org/10.1016/0307-904X\(95\)00123-2](https://doi.org/10.1016/0307-904X(95)00123-2)
- Yadav, R.R., Vinda, R.R., Kumar, N. (1990). One dimensional dispersion in unsteady flow in an adsorbing porous medium: An analytical solution. *Hydrological Processes*, 4(2): 189-196. <https://doi.org/10.1002/hyp.3360040208>
- Srinivasan, V., Clement, T.P. (2008). Analytical solutions for sequentially coupled one-dimensional reactive transport problems—Part II: Special cases, implementation and testing. *Advances in Water Resources*, 31(2): 219-232. <https://doi.org/10.1016/j.advwatres.2007.08.001>
- Yadav, R.R., Jaiswal, D.K. (2012). Two-dimensional solute transport for periodic flow in isotropic porous media: an analytical solution. *Hydrological Processes*, 26(22): 3425-3433. <https://doi.org/10.1002/hyp.8398>
- Jaiswal, D.K., Yadav, R.R., Gulrana (2013). Solute transport under fluctuating groundwater flow in homogeneous finite porous domain. *Hydrogeol. Hydrol. Eng.*, 2(1). <https://doi.org/10.4172/2325-9647.1000103>
- Tamora J., Wadham, C. (2002). Numerical solution of advection-diffusion equations for radial flow. University of Oxford.
- Elfeki, A. (2003). Transient Groundwater Flow in Heterogeneous Geological Formations. (Dept. C). MEJ. Mansoura Engineering Journal, 28(1): 58-67. <http://dx.doi.org/10.21608/bfemu.2021.140807>
- Agusto, F.B., Bamigbola, O.M. (2007). Numerical treatment of the mathematical models for water pollution. *Journal of Mathematics and Statistics*, 3(4): 172-180. <https://doi.org/10.3844/jmssp.2007.172.180>
- Pochai, N. (2009). A numerical computation of a non-dimensional form of stream water quality model with hydrodynamic advection–dispersion–reaction equations. *Nonlinear Analysis: Hybrid Systems*, 3(4): 666-673. <https://doi.org/10.1016/j.nahs.2009.06.003>
- Gurarslan, G., Karahan, H., Alkaya, D., Sari, M., Yasar, M. (2013). Numerical solution of advection-diffusion equation using a sixth-order compact finite difference method. *Mathematical Problems in Engineering*, 2013: 672936. <https://doi.org/10.1155/2013/672936>
- Simmons, C.T., Fenstemaker, T.R., Sharp Jr, J.M. (2001). Variable-density groundwater flow and solute transport in heterogeneous porous media: approaches, resolution and future challenges. *Journal of Contaminant Hydrology*, 52(1-4): 245-275. [https://doi.org/10.1016/S0169-7722\(01\)00160-7](https://doi.org/10.1016/S0169-7722(01)00160-7)
- Freeze, R.A., Cherry, J.A. (1979). *Groundwater*, Prentice-Hall, Inc. Englewood Cliffs, New Jersey.
- Kumar, L. K., Yadav, V., Roy, J. (2022). Numerical Solution for Advection-Dispersion Equation with Uniform and Varying Boundary Conditions. *IIETA*, 9(4): 133-138. <https://doi.org/10.18280/eesrj.090401>
- Ahmad, A., Abdullahi, A., (2024). Numerical Simulation of Lake Pollution Model via Crank-Nicolson Finite Difference Scheme. *IJDM*, 1(20-25) ISSN: 3026-8656. <https://doi.org/10.62054/ijdm/0104.02>