Melting Heat Transfer in Boundary Layer Stagnation-Point Micropolar Fluid Flow towards a Stretching Sheet in Porous Medium with Heat Absorption

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Abstract: The present investigation deals with the study of fluid flow and heat transfer characteristics occurring during the melting process due to a stretching surface in micro-polar fluid with heat absorption. The governing equations representing fluid flow have been transformed into coupled nonlinear ordinary differential equations using similarity transformations. The equations thus obtained have been solved numerically using Runge–Kutta–Fehlberg method with shooting technique. The effects of the melting parameter, micro-polar parameter, heat absorption parameter and stretching parameter on the fluid flow and heat transfer characteristics have been tabulated, illustrated graphically and discussed in detail.

Keywords: Boundary layer, Heat absorption, Melting heat transfer, Micropolar fluid, Stagnation-point, Stretching sheet.

MATHEMATICS SUBJECT CLASSIFICATIONS: 76A05, 76R10, 80A20

Nomenclature

a, c	constant $[m^{-1}]$
c_p	specific heat at constant pressure $[Jkg^{-1}K^{-1}]$
c_s	heat capacity of the solid surface
C_f	skin friction coefficient
f	dimensionless stream function
g	dimensionless angular velocity
j	micro inertia density $[m^2]$
K	micropolar or material parameter
k	thermal conductivity of the fluid $[Wm^{-1}K^{-1}]$
$\mathbf{k_1}$	permeability of porous media
l	reference length
M	melting parameter
N	component of micro-rotation [rad s^{-1}]
n	constant
Nu_x	Nusselt number

Pr	Prandtl number
Q_0	coefficient of heat absorption,
q_w	surface heat flux
Re_x	Reynolds number
T	temperature [K]
T_s	temperature of the solid medium
u, v	velocities in x; y-directions, $[ms^{-1}]$
x, y	axial and perpendicular co-ordinates [m]

Greek Symbols

$\psi(x,y)$	stream function
α	thermal diffusivity
γ	spin-gradient viscosity [N s]
μ	dynamic viscosity [Pa s]
μ_{eff}	effective dynamic viscosity
η	non-dimensional distance
κ	vortex viscosity
υ	kinematic viscosity $[m^2 s^{-1}]$
ρ	fluid density $[kgm^{-3}]$
τ_w	surface heat flux
θ	non-dimensional temperature
λ	latent heat of the fluid
ε	stretching parameter
ε_0	Porosity of porous medium

Subscripts

∞	free stream condition
m	condition at the melting surface

Superscript

$'$	Derivative with respect to η

1. INTRODUCTION

The micropolar fluids are those which contain micro-constituents that can undergo rotation, the presence of which can affect the hydrodynamics of the flow. It has many practical applications, for example, analyzing the behaviour of exotic lubricants, colloidal suspensions, solidification of liquid crystals, extrusion of polymer fluids, cooling of metallic plate in a bath, animal blood, body fluids and so more. The flow and heat transfer characteristic induced by a stretching surface in a non-Newtonian fluid is of great important because it occurs in many manufacturing processes such as in the polymer industry. Eringen [1-3] has introduced the theory of micropolar fluids that is capable to describe those fluids by taking into account the effect arising from local structure and micromotions of the fluid elements.

Das [4] studied the effect of partial slip on steady boundary layer stagnation point flow of an electrically conducting micropolar fluid impinging normally through a shrinking sheet in the presence of a uniform transverse magnetic field. The unsteady MHD boundary layer flow of a micropolar fluid near the stagnation point of a two-dimensional plane surface through a porous medium has been studied by Nadeem et al. [5]. Ishak et al. [6] investigated heat transfer over a stretching surface with variable heat flux in micropolar fluid. Wang [7] investigated the shrinking flow where velocity of boundary layer moves toward a fixed point and they found an exact solution of Navier-Stokes equations. A good list of references for micropolar fluids is available in Łukaszewicz [8]. Tien and Yen [9] investigated the effect of melting on forced convection heat transfer between a melting body and surrounding fluid. Epstein and Cho [10] analyzed the melting heat transfer of the steady laminar flows over a flat plate. The steady laminar boundary layer flow and heat transfer from a warm, laminar liquid flow to a melting surface moving parallel to a constant free stream has been studied by Ishak et al. [11]. Rosali et al. [12] studied micropolar fluid flow towards a permeable stretching /shrinking sheet in a porous medium numerically. Yacob et al. [13] investigated a model to study the heat transfer characteristics occurring during the melting process due to a stretching / shrinking sheet and they have studied the effects of the material parameter, melting parameter and the stretching /shrinking parameter on the velocity, temperature, skin friction coefficient and the local Nusselt number. Cheng and Lin [14] analyzed the melting effect on transient mixed convective heat transfer from a vertical plate in a liquid saturated porous medium. Das [15] analysed the MHD flow and heat transfer from a warm, electrically conducting fluid to melting surface moving parallel to a constant free stream in the presence of thermal radiation and found that fluid temperature and the thermal boundary layer thickness decrease for increasing thermal radiation, melting parameter and magnetic field whereas reverse effect occurs for moving parameter. The effects of thermal radiation using the nonlinear Rosseland approximation are investigated by Cortell [16]. Mahmoud and Waheed [17] presented the effect of slip velocity on the flow and heat transfer for an electrically conducting micropolar fluid through a permeable stretching surface with variable heat flux in the presence of heat generation (absorption) and a transverse magnetic field. They found that local Nusselt number decreased as the heat generation parameter is increased with an increase in the absolute value of the heat absorption parameter. Bataller [18] proposed the effects of viscous dissipation, work due to deformation, internal heat generation (absorption) and thermal radiation. It was shown internal heat generation/absorption enhances or damps the heat transformation. Ravikumar et al. [19] studied unsteady, two-dimensional, laminar, boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing, Rivlin–Ericksen flow fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal buoyancy effect. They observed that heat absorption coefficient increase results a decrease in velocity and temperature. Ahmed and Mohamed [20] studied steady, laminar, hydro-magnetic, simultaneous heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting and heat generating/absorbing fluid over a continuously stretching surface in the presence of the combined effect of Hall currents and mass diffusion of chemical species with first and higher order reactions. They found that for heat source, the velocity and temperature increase while the concentration decreases, however, the opposite behaviour is obtained for heat sink.

In the present work, we consider the boundary layer stagnation- point flow and heat transfer in presence of melting process and heat absorption in a micropolar fluid towards a stretching sheet. We reduced the nonlinear partial differentia equations to a system of ordinary differential equations, by introducing the similarity transformations. The equations thus obtained have been solved numerically using Runge–Kutta–Fehlberg method with shooting technique. The effects of the melting parameter, micropolar parameter heat absorption parameter and stretching parameter on the fluid flow and heat transfer characteristics have been tabulated, illustrated graphically and discussed in detail.

2. MATHEMATICAL FORMULATION

The graphical model of the problem has been given along with flow configuration and coordinate system. The system deals with two dimensional stagnation point steady flow of micropolar fluids towards a stretching surface in porous medium with heat absorption. The velocity of the external flow is $u_e(x) = ax$ and the velocity of the stretching surface is $u_w(x) = cx$, where a and c are positive constant, x is the coordinate measured along the surface. It is also assumed that the temperature of the melting surface and free stream condition is T_m and T_∞, where $T_\infty > T_m$. In addition, the temperature of the solid medium far from the interface is constant and is denoted by T_s where $T_s < T_m$. The viscous dissipation has assumed to be negligible. Under these assumptions, the governing equations representing flow are as follows:

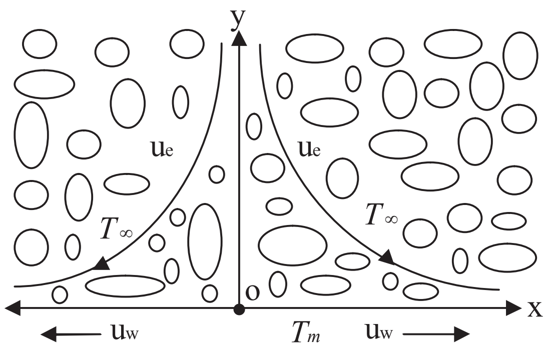

Figure 1 Flow configuration and coordinate system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa\partial N}{\rho\partial y}+\frac{\varepsilon_0\mu_{eff}}{(\rho k_1)(u_e-u)} \tag{2}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{\kappa}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{Q_0}{(\rho c_p)(T-T_m)} \tag{4}$$

with the following boundary conditions:

$$u=u_w(x)=cx\,,\qquad N=-n\frac{\partial u}{\partial y}\,,\qquad T=T_m\,,\qquad at\quad y=0$$

$$u=u_e(x)\rightarrow ax\,,\qquad N\rightarrow 0\,,\qquad T\rightarrow T_\infty\,,\qquad as\quad y\rightarrow\infty$$

and $$k\left(\frac{\partial T}{\partial y}\right)_{y=0}=\rho[\lambda+c_s(T_m-T_s)]v(x,0) \tag{5}$$

Here u and v are the velocity component along the x and y axis respectively. Further, μ is dynamic viscosity, κ is vortex viscosity, σ is electrical conductivity of the fluid, ρ is fluid density, T is fluid temperature, j is micro inertia density, N is microrotation, γ is spin gradient viscosity, α is thermal diffusivity, Q_0 is coefficient of heat absorption, k_1 is permeability of porous media, ε_0 is porosity, μ_{eff} is effective dynamic viscosity, k is the thermal conductivity of fluid, λ is the latent heat of the fluid, c_p is specific heat at constant pressure and c_s is the heat capacity of the solid surface. We note that n is a constant such that $0 \le n \le 1$. The case when $n = 0$, is called strong concentration which indicates that no microrotation near the wall. In case $n = \frac{1}{2}$ it indicates that the vanishing of anti-symmetric part of the stress tensor and denote weak concentration and the case $n = 1$ is used for the modeling of turbulent boundary layer flows by Yacob et al. [13]. $\gamma = \left(\mu + \frac{\kappa}{2}\right)l = \mu\left(1 + \frac{K}{2}\right)l$, where $K = \kappa\frac{\square}{\square}\mu$ is the micropolar or material parameter and $l = \frac{\upsilon}{a}$ as reference length. The total spin N reduces to the angular velocity.

The continuity equation (1) is satisfied by introducing the stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{6}$$

Equations (2), (3) and (4) can be transformed into a set of non-linear ordinary differential equations by using the following similarity transformations:

$$\psi = (a\upsilon)^{1/2} x f(\eta), \quad N = xa\left(\frac{a}{\upsilon}\right)^{\frac{1}{2}} g(\eta), \quad \theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \quad \eta = \left(\frac{a}{\upsilon}\right)^{\frac{1}{2}} y \tag{7}$$

The transformed ordinary differential equations are: $+\Lambda(1 - f')$

$$(1 + K)f''' + ff'' + 1 - f'^2 + Kg' + \Lambda(1 - f') = 0 \tag{8}$$

$$\left(1 + \frac{K}{2}\right)g'' + fg' - f'g - K(2g + f'') = 0 \tag{9}$$

$$\theta'' + \Pr(f\theta' - H\theta) = 0 \tag{10}$$

The boundary conditions (5) become

$$f'(0) = \varepsilon, \quad g(0) = -n f''(0), \quad \Pr f(0) + M\theta'(0) = 0, \quad \theta(0) = 0,$$

$$f'(\infty) \to 1, \quad g(\infty) \to 0, \quad \theta(\infty) \to 1 \tag{11}$$

where primes denote differentiation with respect to η and $Pr = \upsilon/\alpha$ is Prandtl number. The

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$$f'(\infty) \to 1, \qquad g(\infty) \to 0, \qquad \theta(\infty) \to 1. \tag{12}$$

where $\varepsilon = \frac{c}{a}$ is the stretching ($\varepsilon > 0$) parameter, M is the dimensionless melting parameter, H is the heat absorption parameter and Λ is the porous parameter which are defined as

$$M = \frac{C_f(T_\infty - T_m)}{\lambda + C_s(T_m - T_0)}, \qquad H = \frac{Q_0}{a\rho C_p}, \qquad \Lambda = \frac{\varepsilon_0 \mu_{eff}}{a\rho k_1}.$$

(13) The physical parameters of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2}, \qquad Nu_x = \frac{x q_w}{k(T_\infty - T_m)}. \tag{14}$$

where τ_w and q_w are the surface shear stress and the surface heat flux, which are given by

$$\tau_w = \left[(\mu + \kappa)\left(\frac{\partial u}{\partial y}\right)\right]_{y=0}, \qquad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{15}$$

Hence, using (6) we get

$$Re_x^{\frac{1}{2}} C_f = [1 + (1 - n)K]f''(0), \qquad Re_x^{\frac{1}{2}} Nu_x = -\theta'(0) \tag{16}$$

where $Re_x = u_e(x)\frac{x}{\upsilon}$ is the local Reynolds number.

3. METHOD OF SOLUTION

To solve transformed equations (8)–(10) with boundary conditions (11) as an initial value problem, the initial guess values of $f''(0)$, $g'(0)$ and $\theta'(0)$ are chosen and Runge–Kutta–Fehlberg method is applied to obtain a solution. Then the calculated values of $f'(\eta)$, $g(\eta)$ and $\theta(\eta)$ at $\eta = \eta_\infty$ (η_∞ is the sufficient large value of η) are compared with the given boundary conditions $f'(\eta_\infty) = 1$, $g(\eta_\infty) = 0$ and $\theta(\eta_\infty) = 1$. The missing values of $f''(0)$, $g'(0)$ and $\theta'(0)$, for some values of the micropolar parameter K, melting parameter M, heat absorption parameter H and the stretching parameter ε are adjust by shooting method, while the Prandtl number Pr is fixed to unity and we take $n = 0.5$ (weak concentration). We ran the computer program written in MATLAB for different values of η_∞ and for different values of step size

$\Delta\eta$ and found that there is a negligible, change in the velocity, temperature, skin friction coefficient and local Nusselt number for value of $\eta >$ 5. Hence in the present paper the authors have set $\eta_\infty = 5$ and step-size $\Delta\eta = 0.01$.

4. RESULTS AND DISCUSSION

In order to validate the numerical results obtained, we compare our results with reported by Wang [7], Ishak et al. [11], and Yacob et al. [13] as shown in Table 1, and they are found to be in a good agreement. It is seen from Table 2, Figures 2 and 4 that both the melting and material parameters show similar effect on the skin friction coefficient, i.e. increasing K or M is to decrease the values of $f''(0)$, in absolute sense. From Figure 6, it is clear that an increase in heat absorption parameter leads to increase the value of $f''(0)$ in absolute sense. The values of $f''(0)$ are positive when $\varepsilon < 1$, and become negative when $\varepsilon > 1$. Physically, positive value of $f''(0)$ means the fluid exerts a drag force on the solid surface and negative value means the solid surface exerts a drag force on the fluid. The zero skin friction when $\varepsilon = 1$, since for this case the stretching velocity is equal to the free stream velocity.

Table 1 Comparison of values of $f''(0)$ and $-\theta'(0)$ for various values of M, K, ε when $H = \Lambda = 0$.

H	ε	M	K	Yacob et al. [13] $f''(0)$	Yacob et al. [13] $-\theta'(0)$	Present results $f''(0)$	Present results $-\theta'(0)$
	0	0	0	1.232588	-0.570465	1.232586	-0.570465
			1	1.006404	-0.544535	1.006405	-0.5445350
		1	0	1.037003	-0.361961	1.037000	-0.3619620
0			1	0.879324	-0347892	0.879324	-0347892
	0.5	0	0	0.713295	-0.692064	0.713294	-0.692065
			1	0.582403	-0.680176	0.582403	-0.680175
		1	0	0.599090	-0.438971	0.599090	-0.438971
			1	0.506342	-0.432443	0.506341	-0.432443

Table 2 Values of $f''(0)$ and $-\theta'(0)$ for various values of M, K and H with $\varepsilon = 0.5$ and $\Lambda=0$.

ε	M	K	H	$f''(0)$	$-\theta'(0)$
0.5	0	1	0.3	0.582400	-0.420850
0.5	1	1	0.3	0.528650	-0.300421
0.5	2	1	0.3	0.498597	-0.239712
0.5	1	0	0.3	0.631760	-0.306711
0.5	1	2	0.3	0.464470	-0.295800
0.5	1	1	0.0	0.506339	-0.432452
0.5	1	1	0.7	0.506339	-0.186400

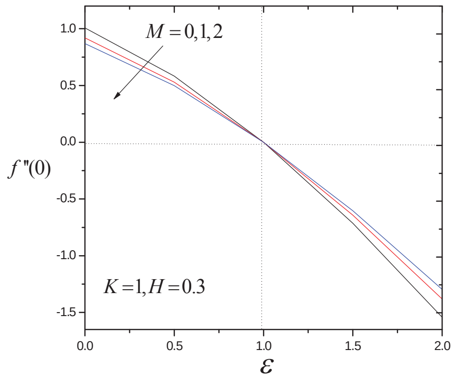

Figure 2 Skin friction coefficient $f''(0)$ with ε for several value of M when Λ=0.

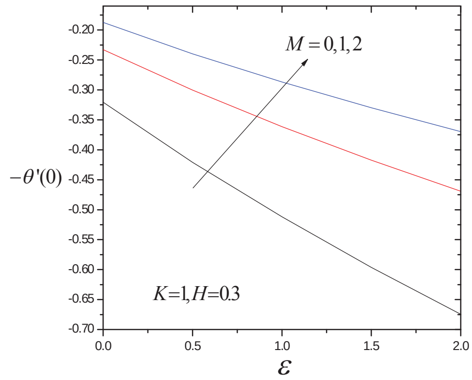

Figure 3 Heat transfer coefficient $-\theta'(0)$ with ε for several value of M when Λ=0.

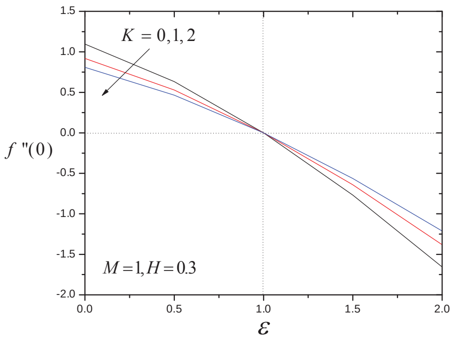

Figure 4 Skin friction coefficient $f''(0)$ with ε for several value of K when Λ=0.

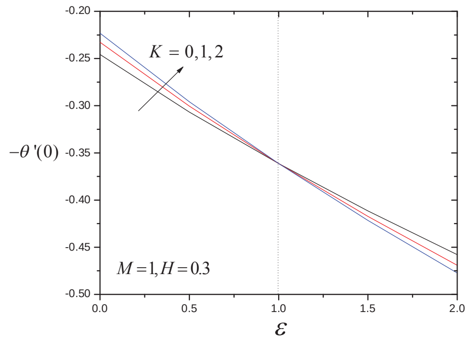

Figure 5 Heat transfer coefficient $-\theta'(0)$ with ε for several value of K when Λ=0.

It is observe from Table 2, Figures 3 and 7 that the heat transfer rate at the fluid-solid interface $|-\theta'(0)|$ decreases as the melting and heat absorption parameters increases. From Figure 5, it is seen that for increasing value of K the heat transfer rate at the fluid-solid interface $|-\theta'(0)|$ decreases for $\varepsilon < 1$, but increases for $\varepsilon > 1$. The negative value of $-\theta'(0)$ presented in Figure 3, 5 and 7 show that the heat is transferred from the warm fluid to cool solid surface.

It is observed from Figures 8 and 10 that the velocity *f '(0)* decreases with the increase in melting parameter *M* and material parameter *K*. From Figure 12 it is clear that the velocity *f '(0)* increases with increasing value of heat absorption parameter *H*. Figures 9, 11 and 13 depicts the effect of *M, K* and *H* respectively on temperature. The temperature decreases with the increase in the values of *M, K* and *H*. Finally, from all the figures (Figs. 8–13), it can be easily seen that the far field boundary conditions (10) are satisfied asymptotically and it signifies the correctness of the numerical scheme used.

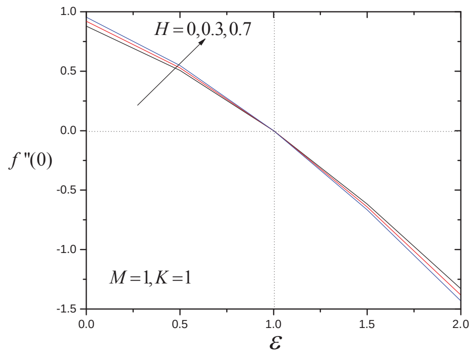

Figure 6 Skin friction coefficient $f''(0)$ with ε for several value of H when Λ=0.

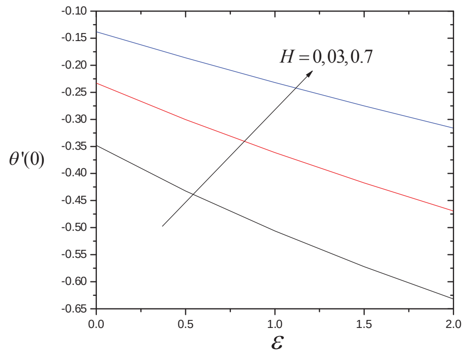

Figure 7 Heat transfer coefficient $-\theta'(0)$ with ε for several value of H when Λ=0.

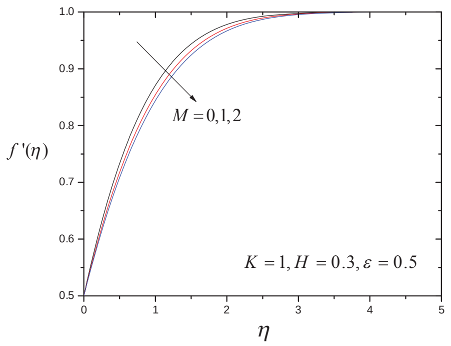

Figure 8 Velocity profiles $f'(\eta)$ for several value of M when Λ=0.

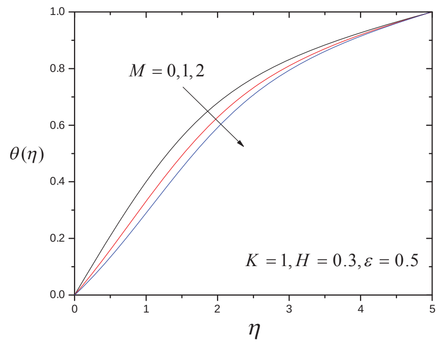

Figure 9 Temperature profile $\theta(\eta)$ for several values of M when Λ=0.

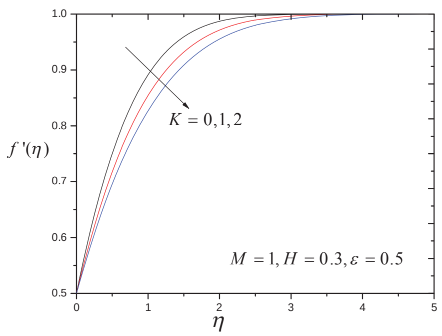

Figure 10 Velocity profiles $f'(\eta)$ for several value of K when Λ=0.

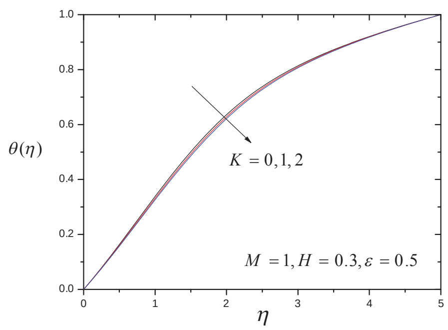

Figure 11 Temperature profile $\theta(\eta)$ for several values of K when Λ=0.

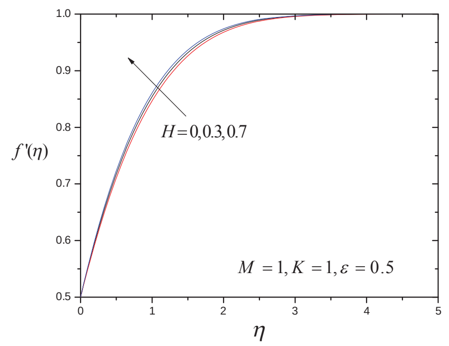

Figure 12 Velocity profiles $f'(\eta)$ for several value of H when Λ=0.

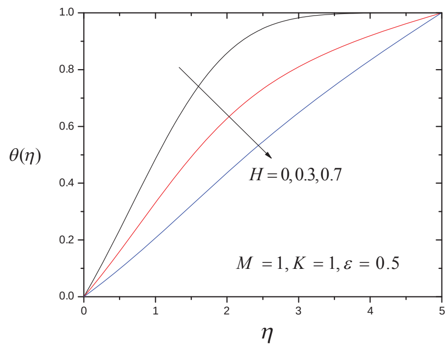

Figure 13 Temperature profile $\theta(\eta)$ for several values of H when Λ=0.

5. CONCLUSION

We have studied the effects of the melting parameter, material parameter, heat absorption parameter and stretching parameter on skin friction coefficient, local Nusselt number (which represents the heat transfer rate at the surface) for the steady laminar boundary layer stagnation point flow and heat transfer from a warm micro-polar fluid to a melting solid surface of the same material. Both the material and the melting parameters decrease the friction and the heat transfer rate at the fluid–solid interface. The heat absorption parameter increases the skin friction coefficient, but decreases the heat transfer rate at the fluid–solid interface.

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