Multi Criteria Decision Making by Using Intuitionistic Fuzzy Preference Relations

Deepa Joshi[1] and Sanjay Kumar[2]

[1,2]Department of Mathematics, Statistics & Computer Science, G. B. Pant University of Agriculture & Technology, Pantnagar-263145, Uttarakhand, India

Email: [1]deepajoshi.6nov@gmail.com, [2]skruhela@hotmail.com

Abstract: In this paper, we present intuitionistic fuzzy preference relations for multi-criteria decision making (MCDM) problem. In proposed method linear programming model is used to obtain optimal weights of criteria, by utilizing criteria weights and accuracy function we select the best one in accordance with the value of weighting function. In order to examine the impact of criteria weights on final ranking sensitivity analysis is performed. A numerical example is also given to clarify the developed approach and to demonstrate its effectiveness.

Keywords: MCDM, intuitionistic fuzzy preference relations, linear programming, accuracy function, sensitivity analysis.

1. INTRODUCTION

Every decision making process produces a final selection of the very best selection after considering a lot of things but the high complexity of socioeconomic environments often makes it difficult for a single decision maker (DM) to consider all the important aspects of some decision problems, therefore in multiple-criteria (or attribute) decision-making problems a decision maker (DM) or (DMs) often needs to select or rank alternatives that are associated with conflicting attributes. This approach often requires the decision makers to provide qualitative and/or quantitative assessments for determining the performance of each alternative with respect to each criterion, and the relative importance of evaluation criteria with respect to the overall objective. These problems arise in many real-world situations. For example, in financial investment, product design, project selection, meal Composition and also plays important role in management problems, investment plan selection, selection of management technique and strategies, selection of operational and computer system. The application of the fuzzy set theory in the field of decision making is justified when the intended goals or their attainment cannot be defined or judged crisply but only as fuzzy sets. But in reality it may not always be certain that the evaluation of membership values, there may be some hesitation degree between membership between membership and non membership. In view that there are many real life situation where due to insufficiency in information availability, intuitionistic fuzzy sets are appropriate to deal with such problems. Intuitionistic fuzzy sets(IFS) introduced by Atanassov [], is characterized by three functions expressing the degree of belongingness, the degree of non belongingness and the degree of hesitation. The concept of IFS is a generalization of fuzzy sets, which currently is used mainly for solving MCDM Problems and group decision making problems when the values of criteria (attributes) of alternatives and/or their weights are intuitionistic fuzzy values. Due to lack of knowledge about background and conflicting nature of alternatives, it is difficult for decision maker to use the exact values to express their preference information about alternatives or criteria, to deal with such cases fuzzy preference relations are used. This relation is based on fuzzy set proposed by Zadeh [9]. To provide the degree that an alternative not priority to another Szmidt and Kacprzyk [6] generalized the fuzzy preference relation to intuitionistic fuzzy preference relations and Xu [8] also introduce the concept of intuitionistic fuzzy preference relation based on intuitionistic fuzzy set proposed by Atanassov [1]. In this paper, we will pay attention to this issue, and investigate the approaches to multi-criteria decision making based on intuitionistic fuzzy preference relations. In order to do that, the remainder of this paper is organized as follows : Section 2 briefly reviews the existing main preference relations. We propose a method for solving multi-criteria decision making (MCDM) problem based on intuitionistic fuzzy preference relations in section 3. In section 4, the proposed method has been implemented on decision making problem to determine the best air-condition system. Sensitivity analysis is also done to see the impact of change of weights on final ranking in section 5. Conclusions are presented in last section.

2. PRELIMINARIES

In these basic definitions of fuzzy set by Zadeh [9], intuitionistic fuzzy set as generalization of fuzzy set by Atanassov [1] are presented. We also described different type of preference relations as follows:

Definition 1. A fuzzy set A in the Universe of discourse X is characterized by membership function $\mu_A : X \to [0,1]$. A fuzzy set A is represented by following order pair :

$$A = \{(x, u_A(x)) : \forall x \in X\} \tag{1}$$

Where u_A is the grade of membership of element x in the set A.

Definition 2. An intuitionistic fuzzy set I on a universe X is defined as an object of the following form :

$$I = \{\langle x, u_I(x), \nu_I(x)\rangle : \forall x \in X\} \tag{2}$$

Where the functions $\mu_I(x) : X \to [0,1]$ and $\nu_I(x) : X \to [0,1]$ represent the degree of membership and degree of non-membership of an element $x \in I \subset X$ respectively.

$\pi_I(x) = 1 - u_I(x) - \nu_I(x)$ is called degree of uncertainty or hesitation of intuitionistic fuzzy set I in X, with the condition $0 \le u_I(x) + \nu_I(x) \le 1$

Definition 3. A fuzzy preference relation R on the set X is represented by a complementary matrix $R=(r_{ij})_{n\times n} \subset X \times X$ for all $i,j = 1,2,.....n$ with

$$r_{ij} \geq 0,\ r_{ij} + r_{ji} = 1,\ r_{ii} = 0.5 \tag{3}$$

where r_{ij} denotes the preference degree of the alternative x_i over x_j.

Definition 4. An intuitionistic preference relation B on a set X is represented by a matrix $B = (b_{ij})_{n\times n} \subset X \times X$ with $b_{ij} = \langle (x_i, x_j), \mu(x_i, x_j), \nu(x_i, x_j)\rangle$ for all i,j=1,2,....,n. For convenience, let $b_{ij} = (\mu_{ij}, \nu_{ij})$, for all i,j = 1, 2, ..., n where b_{ij} is an intuitionistic fuzzy value, composed by the certain degree μ_{ij} to which x_i is preferred to x_j and certain degree ν_{ij} to which x_i is non-preferred to x_i ,and $\pi_{ij}(x)=1-u_{ij}(x)-\nu_{ij}(x)$ is interpreted as the uncertainty degree to which x_i is preferred to x_j and $\pi_{ij}(x)=1-u_{ij}(x)-\nu_{ij}(x)$ is interpreted as the uncertainty degree to which x_i is preferred to x_j.

2.1 Score function

According to Chen and Tan [2], the score function S of x is defined as

$$S(x_{ij})=\mu_{ij}-\nu_{ij} \tag{4}$$

where $S(x_{ij})\in[-1,1]$

The greater the value of score functions the higher the degree of appropriateness that alternative satisfies some criteria.

2.2 Accuracy function

An accuracy function H defined by Hong and Choi [3] to estimate the degree of accuracy of the intuitionistic fuzzy value as

$$H(x_{ij})=\mu_{ij}+\nu_{ij} \tag{5}$$

where $H(x_{ij})\in[0,1]$

the higher degree of accuracy of the degree of membership of the IFS A then it will resulted in the bigger value of $h(x)$. Let $a_1=(u_1,v_1)$, $a_2=(u_2,v_2)$ be two intuitionistic fuzzy values.

Thus,

1. $S(a_1)<S(a_2)$, then $a_1<a_2$
2. $S(a_1)=S(a_2)$, then

   if $H(a_1)<H(a_2)$, then $a_1<a_2$ and

   if $H(a_1)=H(a_2)$, then $a_1=a_2$

3. PROPOSED METHOD

Proposed method uses linear programming model to calculate optimal weights of criteria, given as follows:

$$\max \sum_{i=1}^{m}\{H(x_{ij})*w_j+H(x_{ik})*w_k+H(x_{ip})*w_p\}$$

$$\text{s.t. } w_j^l\le w_j\le w_j^u,$$

$$w_k^l\le w_k\le w_k^u,$$

$$\dots\dots\dots\dots\dots\dots\dots\dots \qquad .$$

$$w_p^l\le w_p\le w_p^u,$$

$$w_j+w_k+\dots\dots+w_p=1 \tag{6}$$

Where w_j , w_k, ..., w_p are respectively weights of the criteria c_i ,c_j ......,c_p

We can solve Eq. (6) by Simplex method.

Then, the degree of suitability to which alternative A_i satisfies the decision maker's requirement can be measured by the weighting function W:

$$W(A_i) = \sum_{i=1}^{m} \{H(x_{ij}) * w_j + H(x_{ik}) * w_k + H(x_{ip}) * w_p\} \tag{7}$$

Where $W(A_i) \in [0,1]$

Step 1: Construct intuitionistic fuzzy relation matrix.

$$X^{(k)} = (x_{ij}^{(k)})_{n \times n}$$

Where i, j, k=1, 2,....n.

Step 2: Aggregating tuplewise by using proposed Aggregation operator:

$$X = \frac{1.x_{11}^{(1)} + 2.x_{11}^{(2)} + 3.x_{11}^{(3)} + ...n.x_{11}^{(n)}}{\frac{n(n+1)}{2}} \tag{8}$$

Where n is the number of decision makers.

Step 3: Accuracy function of each entry of decision matrix is computed.

Step 4: Optimal weights of criteria can be determined by following programming:

$$\max \sum_{i=1}^{m} \{H(x_{ij}) * w_j + H(x_{ik}) * w_k + H(x_{ip}) * w_p\}$$

$$\begin{aligned} \text{s.t. } & w_j^l \le w_j \le w_j^u, \\ & w_k^l \le w_k \le w_k^u, \\ & \dots\dots\dots\dots\dots\dots\dots \\ & w_p^l \le w_p \le w_p^u, \end{aligned}$$

$$w_j + w_k + ... + w_p = 1$$

Step 5: Weighting function of each alternative is determined by (7)

Step 6: Finally rank the alternatives according to descending order of weighting function.

4. IMPLEMENTATION OF PROPOSED METHOD

Consider an air-condition selection problem. Suppose that there are four air-condition system A_1, A_2, A_3 and A_4. These four alternatives are assessed for their performance on the basis of following four criteria.

C_1 = good quality
C_2 = easy to operate
C_3 = economical
C_4 = good service after selling

D_1, D_2, D_3 are three decision makers.

Step 1: Intuitionistic fuzzy preference relation decision matrices according to three decision makers are as follows:

Table 1. Intuitionistic fuzzy preference relation decision matrices

		C_1	C_2	C_3	C_4
	A_1	(0.5, 0.5)	(0.7, 0.1)	(0.6, 0.1)	(0.2, 0.6)
$X^{(1)}$	A_2	(0.6, 0)	(0.5, 0.5)	(0.4, 0.5)	(0.4, 0.5)
	A_3	(0.7, 0.1)	(0.6, 0.2)	(0.5, 0.5)	(0.6, 0.3)
	A_4	(0.8, 0.2)	(0.6, 0.3)	(0.5, 0.3)	(0.5, 0.5)
	A_1	(0.5, 0.5)	(0.6, 0.2)	(0.6, 0.3)	(0.5, 0.2)
$X^{(2)}$	A_2	(0.6, 0.2)	(0.5, 0.5)	(0.7, 0.2)	(0.5, 0.3)
	A_3	(0.7, 0.1)	(0.4, 0.5)	(0.5, 0.5)	(0.5, 0.2)
	A_4	(0.7, 0.1)	(0.6, 0.3)	(0.6, 0.2)	(0.5, 0.5)
	A_1	(0.5, 0.5)	(0.4, 0.3)	(0.6, 0.2)	(0.6, 0.1)
$X^{(3)}$	A_2	(0.7, 0.3)	(0.5, 0.5)	(0.6, 0.2)	(0.5, 0.4)
	A_3	(0.6, 0.2)	(0.6, 0.2)	(0.5, 0.5)	(0.6, 0.2)
	A_4	(0.7, 0.1)	(0.5, 0.2)	(0.5, 0.4)	(0.5, 0.5)

Where $X^{(1)}$, $X^{(2)}$, $X^{(3)}$ are decision matrices according to decision makers D_1, D_2 and D_3 respectively.

Step 2: Aggregating tuplewise each entry of decision matrices by using (7)

Taking no. of decision makers (n) = 3

$$X = \frac{1.x_{11}^{(1)} + 2.x_{11}^{(2)} + 3.x_{11}^{(3)}}{6}$$

So decision maker's weights are (1/6, 2/6, 3/6)

Decision matrix by using proposed aggregation operator is as follows:

Table 2. Decision matrix by aggregating tuplewise

		C_1	C_2	C_3	C_4
	A_1	(0.5, 0.5)	(0.51, 0.23)	(0.6, 0.22)	(0.5, 0.21)
X	A_2	(0.7, 0.21)	(0.5, 0.5)	(0.6, 0.25)	(0.48, 0.38)
	A_3	(0.65, 0.15)	(0.6, 0.3)	(0.5, 0.5)	(0.56, 0.21)
	A_4	(0.72, 0.11)	(0.55, 0.25)	(0.53, 0.35)	(0.5, 0.5)

Step 3: Accuracy function of each entry is given as follows:

$S(x_{11}) = 1$, $S(x_{12}) = 0.74$, $S(x_{13}) = 0.82$, $S(x_{14}) = 0.71$

$S(x_{21}) = 0.91$, $S(x_{22}) = 1$, $S(x_{23}) = 0.85$, $S(x_{24}) = 0.86$

$S(x_{31}) = 0.80$, $S(x_{32}) = 0.9$, $S(x_{33}) = 1$, $S(x_{34}) = 0.77$

$S(x_{41}) = 0.83$, $S(x_{42}) = 0.80$, $S(x_{43}) = 0.88$, $S(x_{44}) = 1$

Step 4: Optimal weights of criteria can be computed by the following programming:

Max $3.54*w_1 + 3.44*w_2 + 3.55*w_3 + 3.34*w_4$

s.t. $0.10 \leq w_1 \leq 0.20$

$0.15 \leq w_2 \leq 0.25$

$0.20 \leq w_3 \leq 0.30$

$0.25 \leq w_4 \leq 0.35$

$w_1 + w_2 + w_3 + w_4 = 1$

Using simplex method to solve the above linear programming, its optimal solution can be obtained as:

$w_1 = 0.20$, $w_2 = 0.25$, $w_3 = 0.30$, $w_4 = 0.25$

Step 5: By applying equation (7) we can get

$W(A_1) = 0.811$, $W(A_2) = 0.897$, $W(A_3) = 0.87$, $W(A_4) = 0.88$

Step 6: Finally rank the alternatives according to descending order of weighting function as follows:
$A_2 > A_4 > A_3 > A_1$

5. SENSITIVITY ANALYSIS

It determines if final answer is stable to changes in the weights of criteria or inputs. In present case sensitivity analysis was performed by examining the impact of criteria weights on the final ranking. We change set of criteria weights four times in this analysis and solve the above linear programming model by Simplex method, we get sets of final rankings as follows:

Weights of criteria	Rank
$W_1 = 0.20$ $W_2 = 0.25$ $W_3 = 0.30$ $W_4 = 0.25$	$A_2 > A_4 > A_3 > A_1$
$W_1 = 0.40$ $W_2 = 0.30$ $W_3 = 0.25$ $W_4 = 0.05$	$A_2 > A_4 > A_3 > A_1$
$W_1 = 0.30$ $W_2 = 0.40$ $W_3 = 0.10$ $W_4 = 0.20$	$A_2 > A_4 > A_3 > A_1$
$W_1 = 0.10$ $W_2 = 0.25$ $W_3 = 0.40$ $W_4 = 0.25$	$A_2 > A_3 > A_4 > A_1$

In above analysis alternative A_2 had the highest ranking.

6. CONCLUSIONS

Preference relations are useful tool in expressing DMs' preferences over alternatives. The existing research mainly focuses on fuzzy preference relations and intuitionistic fuzzy preference relations. Discussed aggregation operator to aggregate decision matrices, is suitable for all types of sets i.e., fuzzy sets, intuitionistic fuzzy sets. To obtain optimal criteria weights linear programming model and accuracy function is used and to select the best alternative weighting function is used. Finally applied proposed approach in decision making problem. To verify the validity of ranking results sensitivity analysis is done by changing criteria weights four times, thus proven to be accurate and suitable and thus, recommended to be used by decision maker.

References

[1] Atanasov, K.: Intuitionistic fuzzy sets. Fuzzy Sets and Systems. 110, 87-96 (1986)

[2] Chen, S. M., Tan, J. M.: Handling multicriteria fuzzy decision making problems based on vague set theory. Fuzzy Sets and Systems. 67, 163-172 (1994)

[3] Hong, D.J.,Choi, C.H.: Multicriteria fuzzy decision making problems based on vague set theory. Fuzzy Sets and Systems. 114, 103-113 (2000)

[4] Herrera, E. Chicl by applying on ana, F., Herrera, F., Alonso, S. (in press).: A group decision making model with incomplete fuzzy preference relations based on additive consistency. IEEE Transactions on System, Man and Cybernetics-Part B.

[5] Lang, C. Hwang, K. Yoon.: Multiple Attribute Decision Making: Methods and Applications. Springer-Verlag, New York. (1981)

[6] Szmidt, E., Kacprzyk, J.: Group Decision Making under Intuitionistic fuzzy preference relations, in proceeding of 7th IPMU Conference. Paris. 172-178 (1998)

[7] Wang, J., Zhang J., Liu, S. Y.: A new score function for fuzzy MCDM based on Vague set theory. Int. journ. Of computational cognition. 4, (2006)

[8] Xu, Z.S.: Intuitionistic preference relations and their applications in group decision making. Information Sciences. 177, 2363-2379 (2007)

[9] Zadeh, L.A.: Fuzzy Sets. Information and Control. 8, 338-353 (1965)